From Spectra to RGB - A Short Introduction to Colorimetry

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1 Introduction

Colorimetry is a physical science that deals with objective, quantitative ways of describing color. Since the impression of color is something subjective — what one person perceives as “light blue”, may look “medium blue” to another — it is not quite clear how to describe and identify different colors. Colorimetry tries (among other things) to connect the notion of a *standard color sample* with the physical concept of light as a form of energy.

Imagine that we want to paint our bedroom and have chosen a suitable shade of yellow. When we buy the paint we have to be able to describe the color in an objective way — if we simply say that we want “yellow” paint we may get the wrong shade. A straightforward way to solve this problem is to use a numbered set of standard color samples such as the PANTONE Color Specifier. We simply select the color that best matches the color we want from the set of samples.

In computer graphics it is often inconvenient and inefficient to use a set of color samples, not only because most of the standard color sets are proprietary and cannot be freely distributed, but also because they often are printed matter and not available in a format suitable for computers. In other words, we need an algorithmic way to define the set of standard colors.

2 The physics of color

The physical phenomenon we call light is electromagnetic energy with wavelengths that varies between approximately 400 nm and 700 nm. Thus, the human eye is sensitive to electromagnetic energy in this interval of the spectrum. We define a *spectral energy distribution* as a function

$$energy = P(\lambda)$$

where λ is the wavelength. In this document, we will only consider wavelenghts that are in the visual part of the spectrum.

Every spectral distribution defines a non-unique color — different distributions can produce the same color impression. Two distributions that look the same are called *metamers*.

It is thought that the retina of the human eye has three kinds of color senors (or *cones*), with peak sensitivities to red, green and blue light. This *tristimulus theory* is attractive since it allows us to think of a color as a positively weighted sum of the *primary colors* red, green and blue. It turns out that a large amount of colors can be reproduced by mixing red, green and blue light.

In 1931 the *Commission Internationale de l'Éclairage (CIE)* defined three standard primaries, called X, Y and Z that can be used to describe all colors the human eye can recognize. The definition of the primaries allows us to compute, in a mathematically well-defined manner, a $[X, Y, Z]$ triplet for all spectral energy distributions. This means that we can use the CIE primaries as our set of "standard colors". That is, we can use them to objectively identify a color in a format suitable for computers.

3 The CIE primaries

The CIE primaries are not color primaries in the sense that they describe the amount of primary colors that should be mixed to reproduce a certain color sample. The Y value is a measurement of the *luminosity*, or brightness, of the color and the X and Z values identify the hue of the color. For a given spectral energy distribution $P(\lambda)$, the corresponding $[X, Y, Z]$ triplet is given by

$$X = k \int_{380}^{780} P(\lambda)\bar{x}(\lambda)d\lambda$$

$$Y = k \int_{380}^{780} P(\lambda)\bar{y}(\lambda)d\lambda$$

$$Z = k \int_{380}^{780} P(\lambda)\bar{z}(\lambda)d\lambda$$

where $\bar{x}$, $\bar{y}$ and $\bar{z}$ are *color matching functions* and k is a constant. The color matching functions were determined experimentally and are available in tabulated form (at 5 nm intervals) from `http://www.cie.co.at/cie/`. In practice, the integrals are approximated by the weighted sums

$$X = k \sum_{\lambda} P(\lambda)\bar{x}(\lambda)\Delta\lambda$$

$$Y = k \sum_{\lambda} P(\lambda)\bar{y}(\lambda)\Delta\lambda$$

$$Z = k \sum_{\lambda} P(\lambda)\bar{z}(\lambda)\Delta\lambda$$

The constant k is usually selected as

$$k = \frac{1}{\int P_w(\lambda)\bar{y}(\lambda)d\lambda}$$

where $P_w(\lambda)$ is the spectral energy distribution of a bright white light. Note that in the discrete case, the $\Delta\lambda$ terms cancel each other so that

$$X = \frac{\sum_{\lambda} P(\lambda)\bar{x}(\lambda)}{\sum_{\lambda} P_w(\lambda)\bar{y}(\lambda)}$$

$$Y = \frac{\sum_{\lambda} P(\lambda)\bar{y}(\lambda)}{\sum_{\lambda} P_w(\lambda)\bar{y}(\lambda)}$$

$$Z = \frac{\sum_{\lambda} P(\lambda)\bar{z}(\lambda)}{\sum_{\lambda} P_w(\lambda)\bar{y}(\lambda)}$$

The value we choose for k is important because it can be used to define the spectral energy distribution function that corresponds to “white”. White light is the sum of light of all colors, so if we start with some $P_w(\lambda)$ that looks white, we can apply filters to $P_w(\lambda)$ to get other colors. A filter is a function $f(\lambda) \in [0,1]$ that we multiply with the spectral energy distribution.

This procedure makes it possible to simulate the interaction between light and surfaces, a common goal in computer graphics. When light arrives at a surface, some is absorbed and some is reflected. If we have a function that describes the fraction of light that is reflected by the surface, we can apply it, as a filter, to the spectral energy distribution of the light source.

The spectral energy distribution most often used to represent white light in computer graphics is called *CIE standard illuminant D* or *illuminant D6500* and is meant to approximate sunlight. It is available in tabulated form from `http://www.cie.co.at/cie/`. An approximate value of k that corresponds to illuminant D is

$$k \approx 9.581530 \cdot 10^{-5}$$

Figure 2 shows a plot of illuminant D. The spectral energy is given as a relative value. For our purposes, it doesn't really matter what quantity the values are measured against (it may be the energy at a given wavelength, for example), since we define the spectral energy distribution as white and apply filter functions in the interval $[0,1]$ to it.

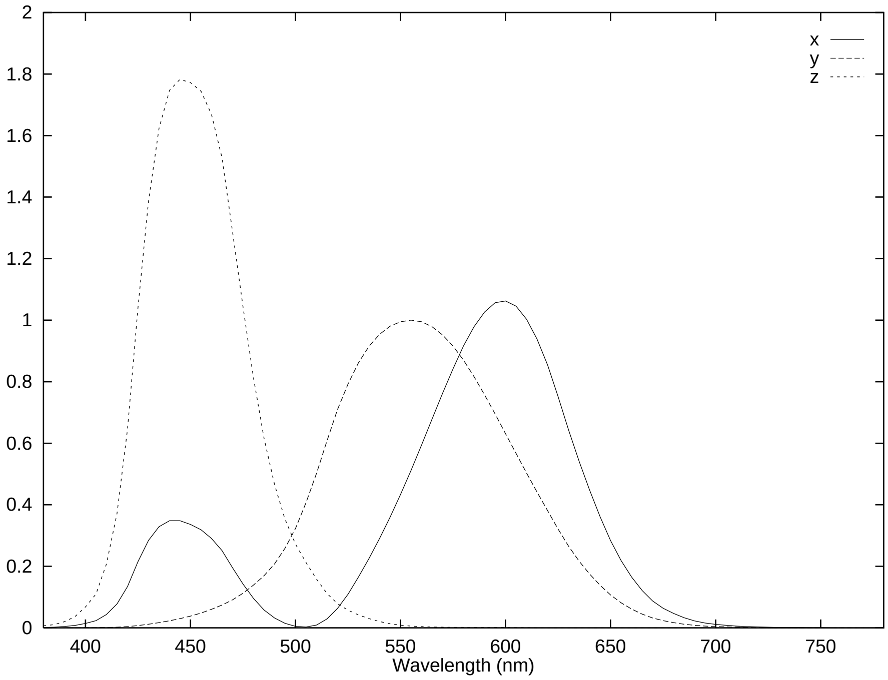

Figure 1: The CIE 1931 Color Matching Functions

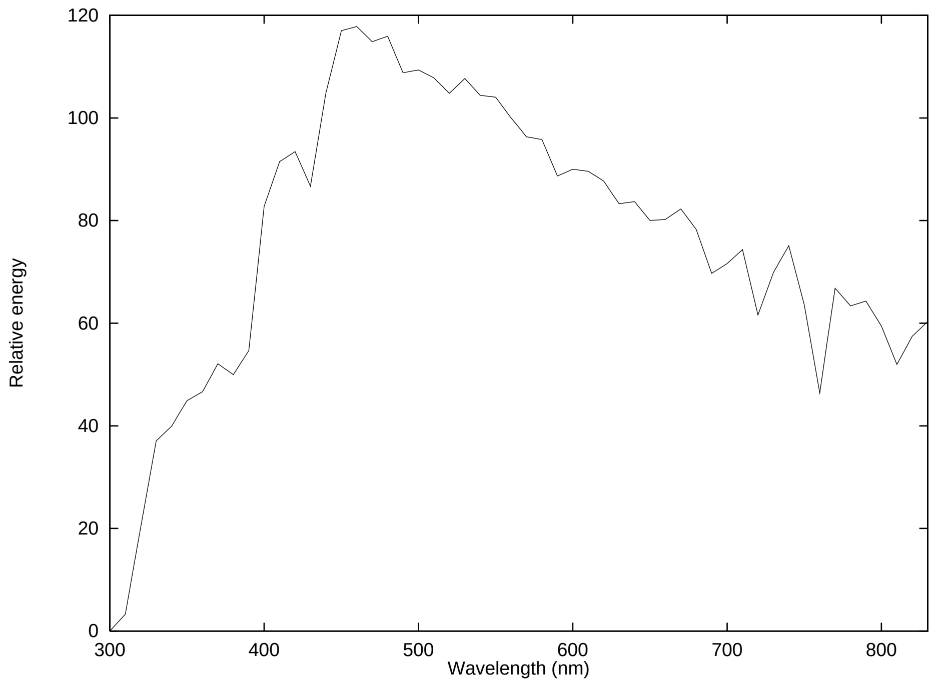

Figure 2: The CIE Standard Illuminant D

4 Chromaticity

Sometimes we are only interested in the hue of a color and not its luminosity. In such cases, the *chromaticity values* of the color is useful. For a given $[X,Y,Z]$ triplet, the corresponding chromaticity values $[x,y,z]$ are defined as

$$x = \frac{X}{X+Y+Z}$$

$$y = \frac{Y}{X+Y+Z}$$

$$z = \frac{Z}{X+Y+Z}$$

Since $x+y+z=1$, we can obtain z from x and y. Given a set of chromaticity values $[x,y]$ and a luminosity Y, we can calculate the corresponding $[X,Y,Z]$ triplet from

$$X = \frac{x}{y}Y$$

$$Y = Y$$

$$Z = \frac{1-x-y}{y}Y$$

The chromaticity values are often used to define *color gamuts*, or color ranges. For example, the colors of the red, green and blue phosphors in computer monitors are usually given as chromaticity values. A set of values $[x_R, y_R]$, $[x_G, y_G]$ and $[x_B, y_B]$ defines a computer monitor gamut — a triangle in $[x,y]$ space. All colors with chromaticity values that lies within the triangle can be reproduced by mixing the colors of the monitor's phosphors — all other colors have to be approximated.

5 The RGB color system

For the purposes of computer graphics, it would be convenient if monitors accepted $[X,Y,Z]$ triplets as input (there exist devices that do). However, the color of a monitor pixel is controlled by applying voltages to three color components: red, green and blue. To transform a $[X,Y,Z]$ triplet to $[R,G,B]$ space, we need the chromaticities of the monitor phosphors and the luminosity and chromaticities of the *white point* of the monitor — that is, the $[x,y,Y]$ values of the point where $R=G=B$. Note that if we use illuminant D and filters as described above, $Y=1$ since the constant k is used to normalize the luminosity to $[0,1]$.

The transformation from $[R,G,B]$ to $[X,Y,Z]$ is

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_R & X_G & X_B \\ Y_R & Y_G & Y_B \\ Z_R & Z_G & Z_B \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

where

$$X_R = x_R C_R \qquad X_G = x_G C_G \qquad X_B = x_B C_B$$

$$Y_R = y_R C_R \qquad Y_G = y_G C_G \qquad Y_B = y_B C_B$$

$$Z_R = (1 - x_R - y_R) C_R \qquad Z_G = (1 - x_G - y_G) C_G \qquad Z_B = (1 - x_B - y_B) C_B$$

$$C_R = \frac{Y_W}{y_W} \frac{x_W (y_G - y_B) - y_W (x_G - x_B) + x_G y_B - x_B y_G}{D}$$

$$C_G = \frac{Y_W}{y_W} \frac{x_W (y_B - y_R) - y_W (x_B - x_R) + x_R y_B - x_B y_R}{D}$$

$$C_B = \frac{Y_W}{y_W} \frac{x_W (y_R - y_G) - y_W (x_R - x_G) + x_R y_G - x_G y_R}{D}$$

$$D = x_R (y_G - y_B) + x_G (y_B - y_R) + x_B (y_R - y_G)$$

In these equations, $[x_R, y_R]$, $[x_G, y_G]$ and $[x_B, y_B]$ are the chromaticity values of the color components of the monitor and $[x_W, y_W, Y_W]$ are the chromaticity values and luminosity of the white point of the monitor. The transformation from $[X,Y,Z]$ space to $[R,G,B]$ space is given by the inverse of the matrix. If a $[R,G,B]$ triplet is outside the monitor gamut, at least one of the values is < 0.

The gamut of a monitor is usually available from the manufacturer, but if we want to display colors on a monitor with an unknown gamut, we can use the primaries for high-definition television (HDTV), which are internationally agreed upon and match most contemporary monitors very well. The chromaticities that correspond to illuminant D are listed in table 1.

For HDTV primaries, the transformation matrices are

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 3.240479 & -1.537150 & -0.498535 \\ -0.969256 & 1.875992 & 0.041556 \\ 0.055648 & -0.204043 & 1.057311 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.412453 & 0.357580 & 0.180423 \\ 0.212671 & 0.715160 & 0.072169 \\ 0.019334 & 0.119193 & 0.950227 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

If a color C with chromaticities $[x_C, y_C]$ is outside the monitor gamut, it is necessary to approximate it with a color that lies inside the gamut. The best way to do this is still an open area of research, but for most applications the color can be approximated by calculating the intersection of the line from $[x_W, y_W]$ to $[x_C, y_C]$ with the triangle that defines the gamut. The algorithm is

```
#define CLAMP(v,l,h) ((v)<(l) ? (l) : (v) > (h) ? (h) : (v)

[wr, wg, wb] = RGB values of monitor white point
[r, g, b] = input color (assumed to be outside monitor gamut)

if (r < g && r < b)
  p = wr / (wr - r);
else {
  if (g < r && g < b)
    p = wg / (wg - g);
  else
    p = wb / (wb - b);
}

Rclamp = CLAMP(wr + p * (r - wr), 0, 1);
Gclamp = CLAMP(wg + p * (g - wg), 0, 1);
Bclamp = CLAMP(wb + p * (b - wb), 0, 1);
```

6 Gamma correction

The voltages presented to a computer monitor control the intensities of the red, green and blue primaries, but in a nonlinear manner — the intensity is approximately the applied voltage raised to the 2.5 power. The $[R, G, B]$ values we obtain by using the methods presented above, however, are linear. We can convert a linear $[R, G, B]$ triplet to a triplet $[R_{volt}, G_{volt}, B_{volt}]$ suitable for the HDTV monitor type by using

	R	G	B	white
x	0.640	0.300	0.150	0.3127
y	0.330	0.600	0.060	0.3290

Table 1: Chromaticity values for HDTV primaries corresponding to illuminant D

$$R_{volt} = \begin{cases} 4.5R & \text{if } R \leq 0.018 \\ 1.099R^{0.45} - 0.099 & \text{if } R > 0.018 \end{cases}$$

$$G_{volt} = \begin{cases} 4.5G & \text{if } G \leq 0.018 \\ 1.099G^{0.45} - 0.099 & \text{if } G > 0.018 \end{cases}$$

$$B_{volt} = \begin{cases} 4.5B & \text{if } B \leq 0.018 \\ 1.099B^{0.45} - 0.099 & \text{if } B > 0.018 \end{cases}$$

This procedure is known as *gamma correction*. For more information, see the Gamma FAQ.

References

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